

STATISTICAL QUALITY CONTROL (SQC) (IEng 5241)

Chapter Two

Quality Control Techniques

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Introduction:

What is Quality Control?

Quality control is the **engineering & management activity** by which we measure the **quality characteristics** of the product, compare them with **specifications or requirements**, and take **appropriate remedial action** whenever there is a difference between the **actual performance** and the **standard**.

Introduction:

Quality control is defined as: “A system of methods for the **cost effective** provision of products or services whose **quality is good for the purchaser's requirements**”.

(Ishikawa)

Introduction:

“Quality control consists of developing, designing, producing, marketing and servicing products and services with optimum cost-effectiveness and usefulness, which Customers will purchase with satisfaction”.

(Ishikawa)

Introduction:

To produce products/services that consumers will buy happily;

- **Quality dimensions**,
- **Costs** (i.e. sales price and profit),
- **Delivery** (i.e. production volumes and sales volumes), and
- **Safety** (including social and environmental factors) must be comprehensively controlled.

Introduction:

Quality products/service can be achieved through full use of **variety of techniques** such as:

- Statistical and technical methods,
- Standards and regulations,
- Computer methods, automatic control, and
- Industrial engineering techniques and market research.

Quality Control Evolution

- The objective of QC function at
operators level,
foremen level,
inspectors level, or
managers level

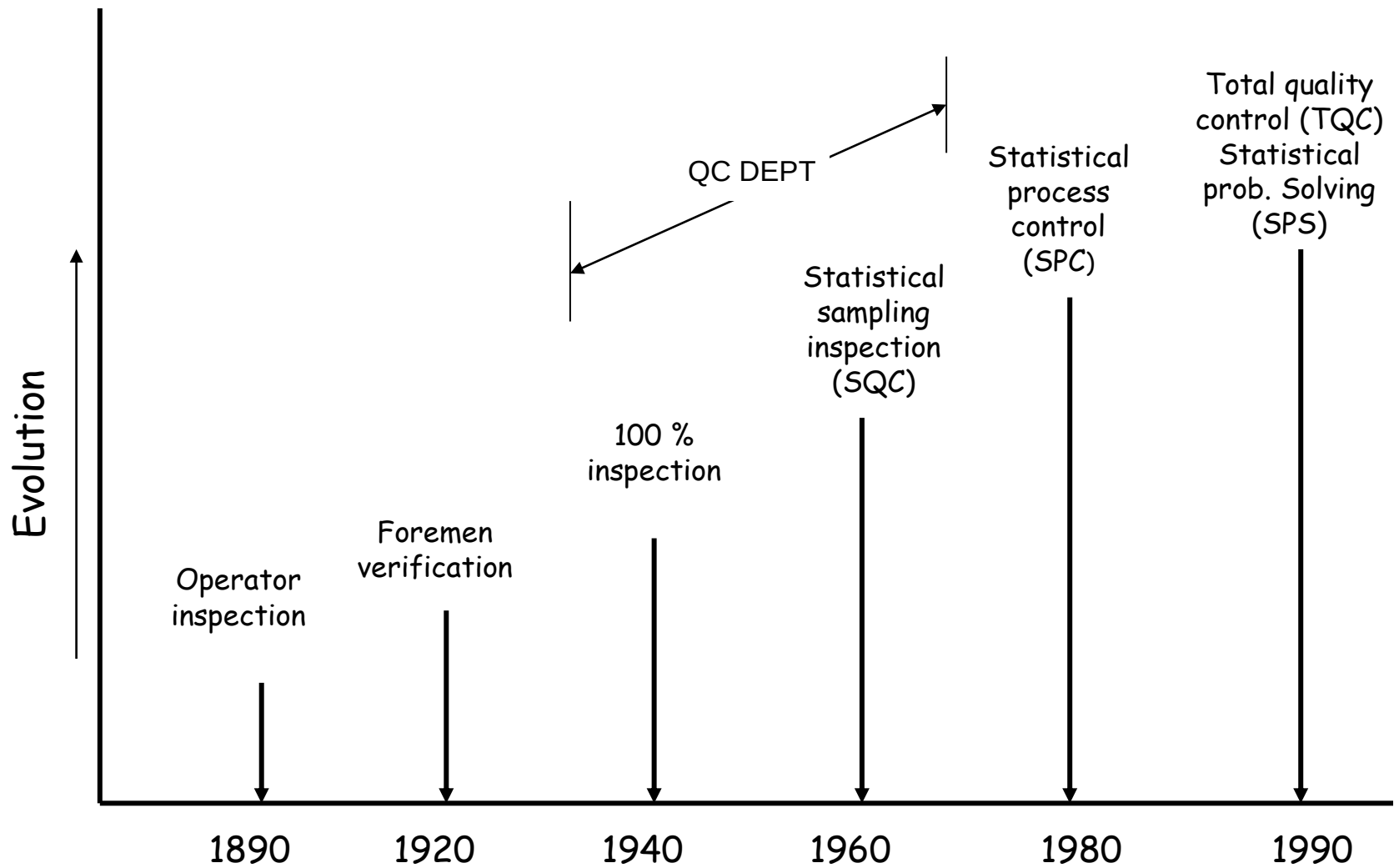
is to involve with operational techniques and activities carried both at monitoring a process and at eliminating causes of unsatisfactory performance.

1. **Operators quality control:** this was inherent in the manufacturing jobs upto the end of 19th century.
 - Under this system, **one worker** or a **very small number of workers** were **responsible** for the manufacture of the entire product and therefore, **each worker could totally control the quality of his/her work.**

2. Foremen quality control: this kind of QC is due to the advent of our modern factory concept, in which many individuals performing a similar task were grouped so that they could be directed by a foremen who then assumed responsibility for the quality of the their work.

3. **Inspectors quality control:** Manufacturing system became more complex during the world war-I, involving large number of workers reporting to each production foreman. As a result, the first full time inspector appeared on the scene initiating the 3rd step known as inspection QC.

4. **Managers quality control**: in effect, this is an extension of the inspection phase and boiled down to making the big inspection organization more efficient. The most significant contribution of SQC is that it provides sampling inspection rather than 100% inspection. The task of quality control, however, remains restricted to production areas only.



The Need for Quality Control

- If a defective product enters in the market, it will cause:
 - customer dissatisfaction,
 - unnecessary expenditure for warranty, and
 - poor product salability.
- Having a quality product increases market share, resulting in better profits.

Statistical Process Control (SPC) Techniques

- ❖ Statistical Process Control (SPC) is an analytical decision making tool which allows you to see when a process is working correctly and when it is not. Variation is present in any process, deciding when the variation is natural and when it needs correction is the key to quality control.

Statistical Process Control (SPC) Tools

The key process monitoring and investigating tools include:

- Histograms,
- Check sheets,
- Pareto charts,
- Cause and effect diagrams,
- Scatter diagrams, and
- Control charts.
- Defect concentration.

Histogram

The histogram is a bar chart showing a distribution of variables.

This tool helps to identify the cause of problems in a process by shape of the distribution as well as the width of the distribution.

Histogram (cont'd)

- The histogram clearly portrays information on location, spread, and shape regarding the functioning of the physical process.
- It can also help to suggest both the nature of, and possible improvements for, the physical mechanisms at work in the process.

Histogram (cont'd)

Steps in constructing Histogram

a) How to Make Frequency Tables

Step 1: Calculate the range (R)

$R = (\text{the largest observed value}) - (\text{the smallest observed value})$.

Obtain the largest and the smallest of observed values and calculate the range R .

Histogram (cont'd)

Step 2: Determine the class interval

The class interval is determined so that the **range**, which includes the maximum and the minimum of values, is **divided in to intervals of equal breadth**.

Histogram (cont'd)

Step 3: Prepare the frequency table

Prepare a table in which the class, midpoint, frequency marks, etc., can be recorded.

Histogram (cont'd)

Step 4: Determine the class boundaries

Determine the boundaries of the intervals so that they include the smallest and the largest of values, and write these down on the frequency table.

Histogram (cont'd)

To obtain the interval breadth, divide R by 1, 2, or 5 (or 10, 20, 50; 0.1, 0.2, 0.5 etc) so as to obtain from 5 to 20 class intervals of equal breadth. When there are two possibilities, use the narrower intervals if the number of measured values is 100 or over and the wider interval, if there are 99 or less observed values.

Histogram (cont'd)

Step 5: Calculate the mid point of the class

Using the following equation, calculate the mid-point of class, and write this down on the frequency table.

$$\text{Mid - point of the first class} = \frac{\text{Sum of the upper \& lower boundaries of the first class}}{2}$$

Similarly,

$$\text{Mid - point of the second class} = \frac{\text{Sum of the upper \& lower boundaries of the second class}}{2}$$

Histogram (cont'd)

The midpoints of the second class, the third class, and so on.

Midpoints may also be determined as follows:

- Midpoint of the second class = midpoint of the first class + class interval
- Midpoint of the third class = midpoint of the second class + class interval

•
•
•

and so on.

Histogram (cont'd)

Step 6: Obtain the frequencies

Read the observed values one by one and record the frequencies falling in each class using tally marks, in group of five.

Histogram (cont'd)

Example 6.1

Akaki Spare Parts and Hand Tools Share Company wants to investigate the distribution of the diameters of shafts produced in a grinding process, the diameter of 90 shafts are measured as shown in the following table. Draw a histogram using these data.

Table 6.1: Sample and Result of Measurement

Sample Number	Results of measurements									
1-10	2.510	2.517	2.522	2.522	2.510	2.511	2.519	2.532	2.543	2.525
11-20	2.527	2.536	2.506	2.541	2.512	2.515	2.521	2.536	2.529	2.524
21-30	2.529	2.523	2.523	2.523	2.519	2.528	2.543	2.538	2.518	2.534
31-40	2.520	2.514	2.512	2.534	2.526	2.530	2.532	2.526	2.523	2.520
41-50	2.535	2.523	2.526	2.525	2.523	2.522	2.502	2.530	2.522	2.514
51-60	2.533	2.510	2.542	2.524	2.530	2.521	2.522	2.53	2.540	2.528
61-70	2.525	2.515	2.520	2.519	2.526	2.527	2.522	2.542	2.540	2.528
71-80	2.531	2.545	2.524	2.522	2.520	2.519	2.519	2.529	2.522	2.513
81-90	2.518	2.527	2.511	2.519	2.531	2.527	2.529	2.528	2.519	2.521

Solution:

Step 1: Calculate R

R is obtained from the largest and the smallest of observed values. Therefore; from the table 6.1:

The largest value is 2.545

The smallest value is 2.502

Thus, $R = 2.545 - 2.502 = 0.043$

Step 2: Determine the class interval

$0.043/0.002 = 21.5$, and we can make this 22 by rounding up to the nearest integer

$0.043/0.005 = 8.6$, and we can make this 9 by rounding up to the nearest integer

$0.043/0.010 = 4.3$, and we can make this 4 by rounding down to the nearest integer.

Thus, the class interval is determined as 0.005, since this gives a number of intervals between 5 and 20.

Histogram (cont'd)

Step 3: Prepare the frequency table

Prepare a table as shown in R Table 6.2

Step 4: Determine the class boundaries

The boundaries of the first class should be determined as 2.5005 and 2.5055 so that the class includes the smallest value 2.50; the boundaries of the second class should be determined as 2.5055-2.515, and so on. Record these on frequency table.

Step 5: Calculate the mid-point of class

Mid point of the first class

$$= \frac{2.5005 + 2.5055}{2} = 2.503$$

**Mid point of the second class
and so on.**

$$= \frac{2.5055 + 2.5105}{2} = 2.508$$

Histogram (cont'd)

Step 6: obtain the frequencies

Record the frequencies. (see table 6.2)

Table 6.2 Frequency Table

	Class	Mid-point of class x	Frequency mark (tally)	Frequency f
1	2.5005-2.5055	2.503	/	1
2	2.5055-2.5105	2.508	////	4
3	2.5105-2.5155	2.513	///// ////	9
4	2.5155-2.5205	2.518	///// ///// ////	14
5	2.5205-2.5255	2.523	///// ///// ///// ///// //	22
6	2.5255-2.5305	2.528	///// ///// ///// ////	19
7	2.5305-2.5355	2.533	///// /////	10
8	2.5355-2.5405	2.5338	/////	5
9	2.5405-2.5455	2.543	///// /	6
	Total			90

Histogram (cont'd)

b) How to make a Histogram

Step 1:

On a sheet of squared paper, mark the horizontal axis with a scale. The scale should not be on the base of class interval but it is better to be on the base of measurement of data, (e.g. 10 grams correspond to 10 mm).

Histogram (cont'd)

Step 2:

Make the left-hand vertical axis with a frequency scale, and, if necessary, draw the right-hand axis and mark it with a relative frequency scales.

Histogram (cont'd)

Step 3:

Make the horizontal scale with the class boundary values.

Step 4:

Using the class interval as a base line, draw a rectangle whose height corresponds with the frequency in that class.

Histogram (cont'd)

Step 5:

Draw a line on the histogram to represent the mean, and also draw a line representing the specification limit, if any.

Step 6:

In a blank area of the histogram (Figure below), **note the history of the data.**

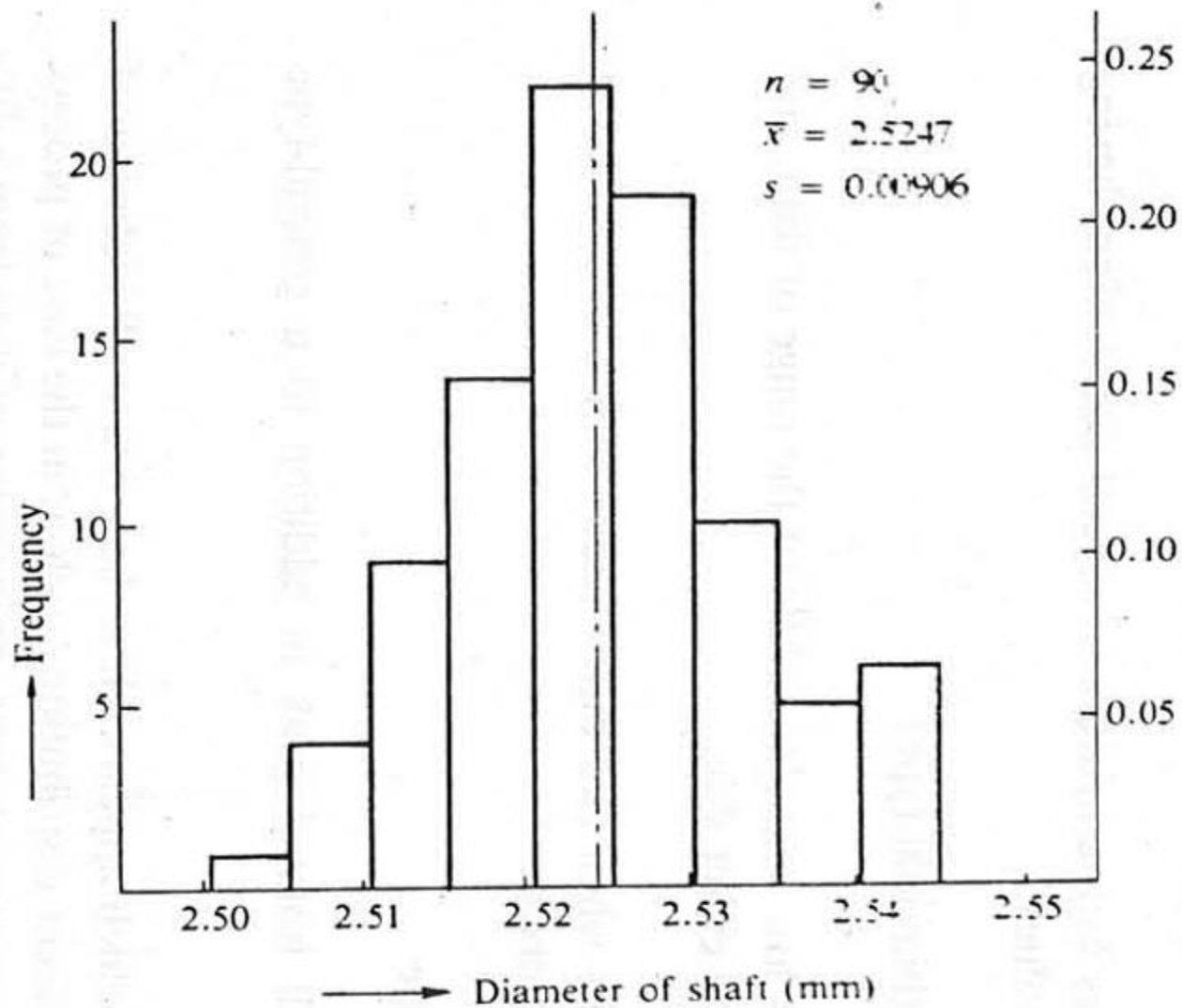
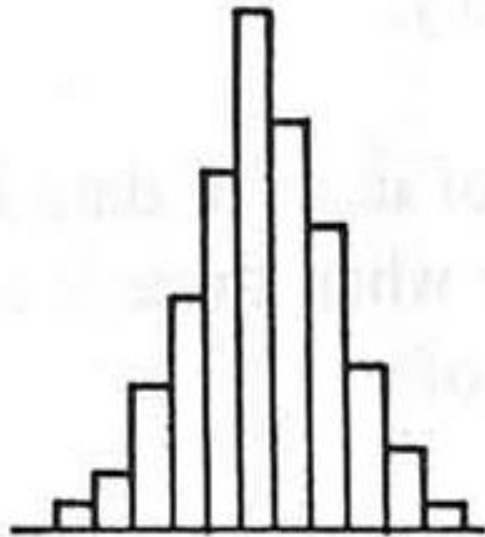


Figure: Histogram for the above example

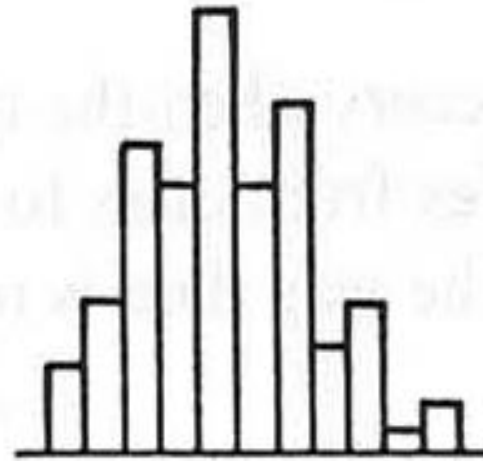
Types of Histograms

General type: it is symmetrical or bell-shaped. The mean value of the histogram is in the middle of the range.



a) General type

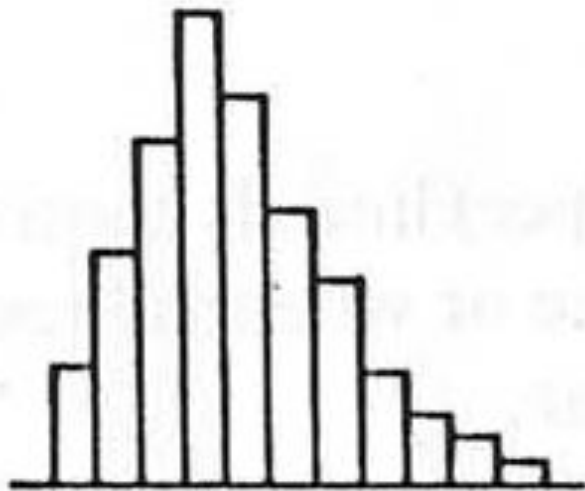
Comb type: This shape occurs when the number of units of data included in the class varies from class to class.



b) Comb type

Positively skew type:

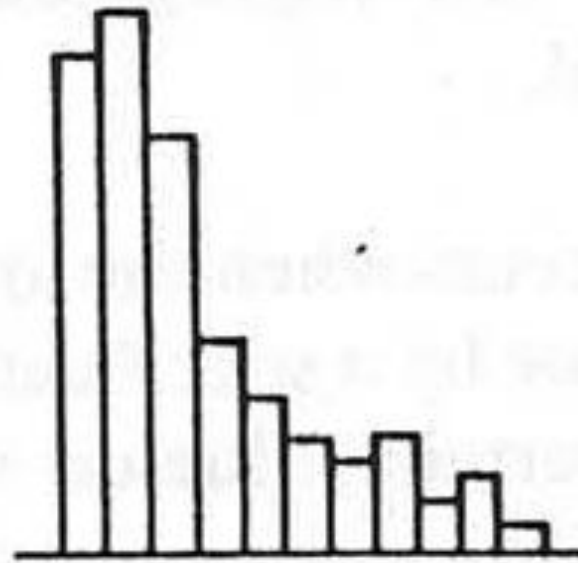
(Negatively skew type): The mean value of the histogram is located to the left (right) of the center of the range.



c) Positively skew type

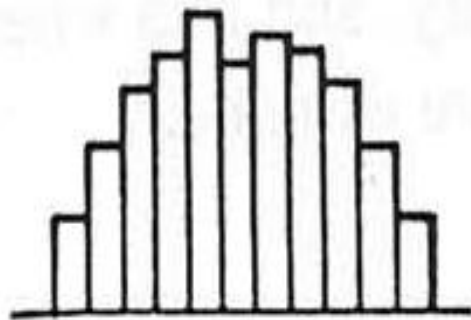
Left hand precipice type

(right hand precipice type): The mean value of the histogram is located for to the left (right) of the center of range.



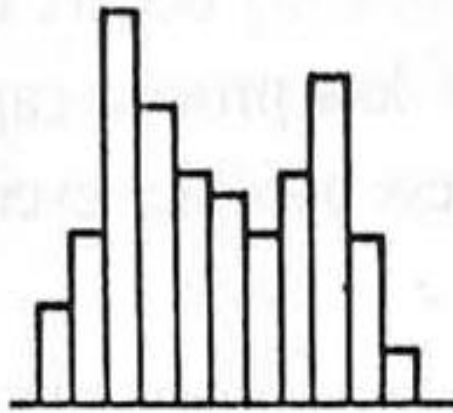
d) Left-hand precipice type

Plateau type: The frequency in each class forms a plateau because the classes have more or less the same frequency except for those at the ends.



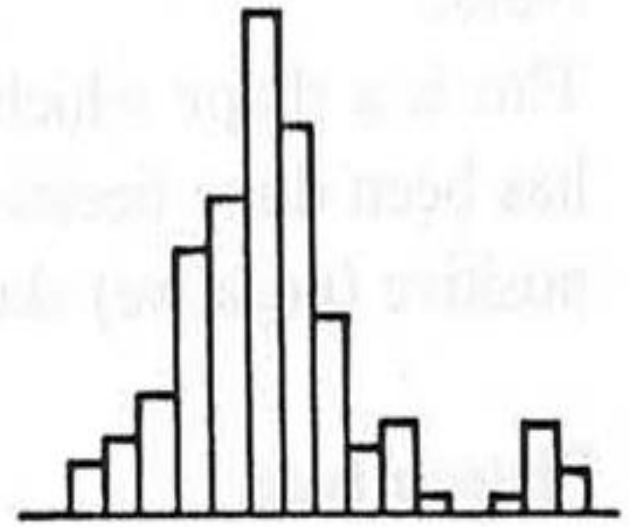
e) Plateau type

Twin-peak type : (bimodal type): The frequency is low near the middle of the range of data, and there is a peak on either side.



f) Twin-peak type

Isolate-peak type: There is a small isolated peak in addition to a general type histogram.



g) Isolated-peak type

2. Check Sheet

A check sheet is a **paper form** on which items to be **checked have been printed already** so that data can be collected easily and concisely.

Its **main purposes** are:

- To make data-gathering easy
- To arrange data automatically so that they can be used easily later on.

Defective item check sheet

Check Sheet		
Product: _____	Date: _____	
Manufacturing stage: final insp. _____	Section: _____	
Type of defect: scar, incomplete crack, misshapen _____	Inspector's name: _____	
Total no. inspected: 1525 _____	Lot no.: _____	
Remarks: all items inspected _____	Order no.: _____	

Type	Check	Subtotal
Surface scars	 	17
Cracks	 	11
Incomplete	 	26
Misshapen	 	3
Others	 	5
	Total:	62
Total rejects	 	42

3. Pareto Analysis

Pareto (80/20 principle)

Quality problems appear in the form of loss (defective items and their cost). It is extremely important to clarify the distribution pattern of the loss. Most of the loss will be due to a very few types of defect, and these defects can be attributed to a very small number of causes. Thus, if the causes of these **Vital few defects** are identified, we can eliminate almost all the losses by concentrating on these particular causes, leaving aside the other **Trivial many defects** for the time being. By using the Pareto diagram, we can solve this type of problem efficiently.

A Pareto diagram is a **bar graph** used to **arrange information** in such a way that **priorities for process improvement** can be established.

Pareto Diagram (cont'd)

Pareto diagram is used for:

- 1.To display the relative importance of data.
- 2.To direct efforts to the biggest improvement opportunity by highlighting the vital few in contrasts to the useful trivial many.

Pareto Diagram (cont'd)

Steps to construct a Pareto diagram:

Step 1: Determine the categories and the units for comparison of the data, such as frequency, cost, or time.

Pareto Diagram (cont'd)

Step 2:

Total the raw data in each category, then determine the grand total by adding the totals of each category.

Step 3:

Re-order the categories from largest to smallest.

Pareto Diagram (cont'd)

Step 4:

Determine the cumulative percent of each category (i.e., the sum of each category plus all categories that precede it in the rank order, divided by the grand total and multiplied by 100).

Pareto Diagram (cont'd)

Step 5:

Draw and label the left-hand vertical axis with the unit of comparison, such as frequency, cost or time.

Step 6:

Draw and label the horizontal axis with the categories. List from left to right in rank order.

Pareto Diagram (cont'd)

Step 7:

Draw and label the right-hand vertical axis from 0 to 100 percent. The 100 percent should line up with the grand total on the left-hand vertical axis.

Step 8:

Beginning with the largest category, draw in bars for each category representing the total for that category.

Pareto Diagram (cont'd)

Step 9: Draw a line graph beginning at the right-hand corner of the first bar to represent the cumulative percent for each category as measured on the right-hand axis.

Step 10: Write any necessary items on the diagram.

Step 11: Analyze the chart. Usually the top 20% of the categories will comprise roughly 80% of the cumulative total.

Pareto Diagram (cont'd)

Example

The following table shows the different types of defect and the total number of items that are occurred on selected products in an ideal company ABC. Use the Pareto analysis to determine the vital few cause, which results the majority of the problem.

Table 6.3 Number of defects observed

Type of Defect	Number of Defects
Crack	10
Scratch	42
Stain	6
Strain	104
Gap	4
Pinhole	20
Others	14
Total	200

Pareto Diagram (cont'd)

Solution:

Step 1:

1. Decide what problems are to be investigated and how to collect the data.
2. Decide what kind of problems you want to investigate.

Example: Defective items, losses in monetary terms, accidents occurring.

Pareto Diagram (cont'd)

3. Decide what data will be necessary and how to classify them. *Example:* By type of defect, location, process, machine, worker, method.

Note: Summarize items appearing infrequently under the heading "others."

4. Determine the method of collecting the data and the period during which it is to be collected.

Note: Use of an investigation form is recommended.

Pareto Diagram (cont'd)

Step 2:

Design a data tally sheet listing the items, with space to record their totals

Step 3:

Make a Pareto diagram data sheet listing the items, their individual totals, cumulative totals, percentages of overall total, and cumulative percentages (Table 6.4).

Pareto Diagram (cont'd)

Step 4:

Arrange the items in the order of quantity, and fill out the data sheet.

Note: The item "others" should be placed in the last line, no matter how large it is. This is because it is composed of a group of items each of which is smaller than the smallest item listed individually.

Table : Data Sheet for Pareto Diagram

Type of Defects	Number of Defects	Cumulative Total	Percentage of overall Total	Cumulative Percentage
Strain	104	104	52	52
Scratch	42	146	21	73
Pinhole	20	166	10	83
Crack	10	176	5	88
Stain	6	182	3	91
Gap	4	186	2	93
Others	14	200	7	100
Total	200	-	100	-

Pareto Diagram (cont'd)

Step 5:

Draw Left-hand vertical axis and mark this axis with a scale from 0 to the overall total two vertical axes and a horizontal axis.

Step 6:

Draw horizontal axis, and divide this axis into the number of intervals to the number of items classified.

Pareto Diagram (cont'd)

Step 7: Draw Right-hand vertical axis and mark this axis with a scale from 0 % to 100 %.

Step 8: Construct a bar diagram.

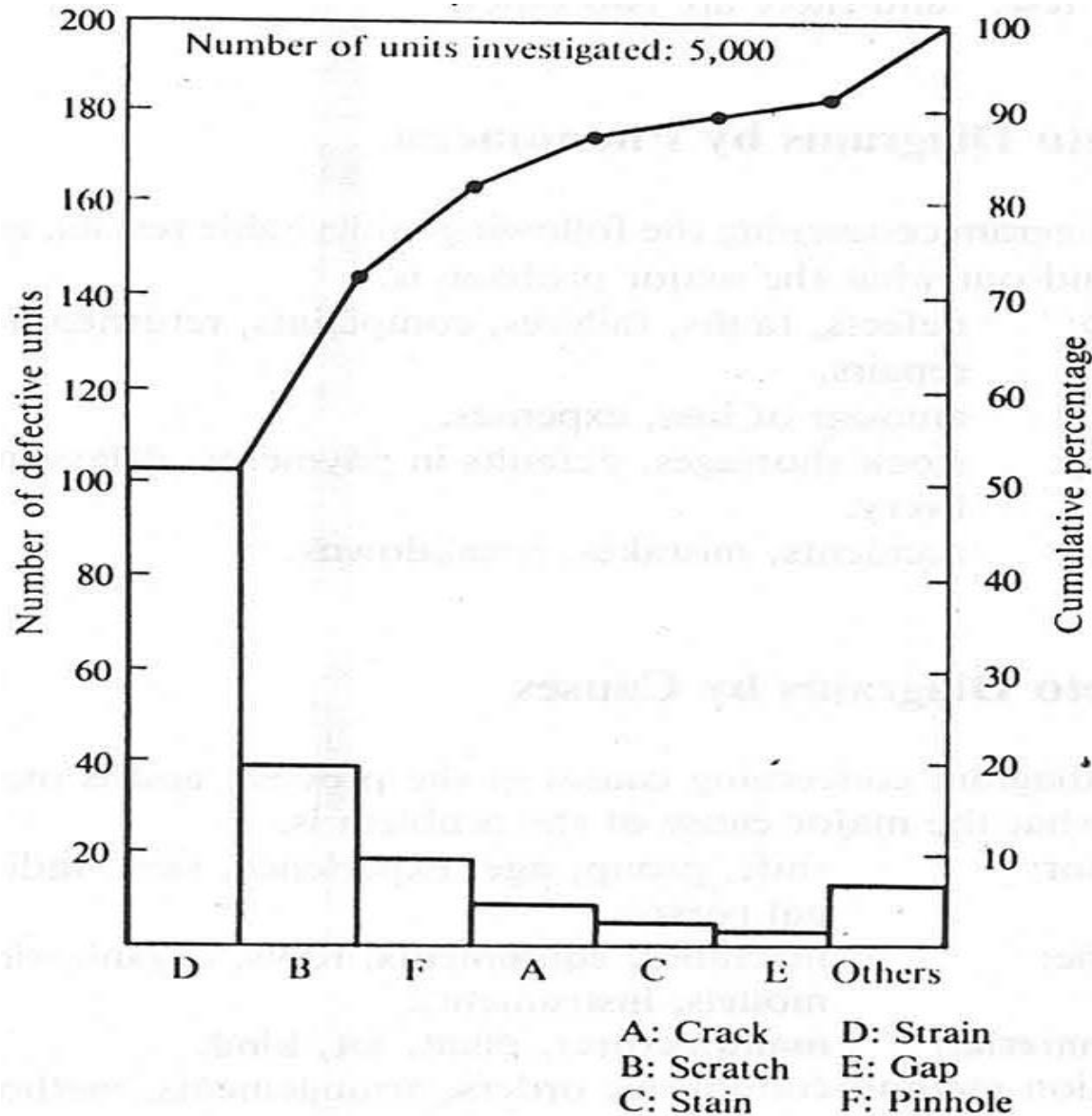
Step 9: Draw the cumulative curve (Pareto curve) as shown in figure 6.2. Mark the cumulative values (cumulative total or cumulative percentage), above the right-hand intervals of each item, and connect the points by a solid line.

Pareto Diagram (cont'd)

Step 10: Write any necessary items on the diagram.

1. Items concerning the diagram as title,
significant quantities, units, name of drawer
2. Items concerning the data as period, subject
and place of investigations, total number of
data etc.

Step 11: Analyze the chart.



**Figure : Pareto
Diagram by
Defective
Items**

4. Cause-and-Effect Diagram

- A Cause-and-Effect Diagram is a tool that helps identify, sort, and display possible causes of a specific problem or quality characteristic.
- The diagram graphically illustrates the relationship between a given outcome and all the factors that influence the outcome.

Cause-and-Effect (cont'd)

It is used when we need to:

- Identify the possible root causes, the basic reasons, for a specific effect, problem, or condition.
- Sort out and relate some of the interactions among the factors affecting a particular process or effect.
- Analyze existing problems so that corrective action can be taken.

Cause-and-Effect (cont'd)

Some of the benefits of constructing a Cause-and-Effect Diagram are that it:

1. Helps determine the root causes of a problem or quality .
2. Encourages group participation.
3. Uses an orderly, easy-to-read format .

Cause-and-Effect (cont'd)

- 4.Indicates possible causes of variation in a process.**
- 5.Increases knowledge of the process by helping everyone to learn more about the factors at work and how they relate.**
- 6.Identifies areas where data should be collected for further study.**

Developing a Cause-and-Effect Diagram

The steps for constructing and analyzing a Cause-and-Effect Diagram are :

Step 1: Identify and clearly define the outcome or effect to be analyzed.

1. Decide on the effect to be examined.
2. Use operational definitions.
3. Remember, an effect may be positive (an objective) or negative (a problem),

Cause-and-Effect (cont'd)

Step 2: Using a chart pack positioned so that everyone can see it, draw the spine and create the effect box.

- 1. Draw a horizontal arrow pointing to the right. This is the spine.**
- 2. To the right of the arrow, write a brief description of the effect or outcome, which results from the process.**

Cause-and-Effect (cont'd)

**Step 3: Identify the main causes
contributing to the effect being studied**

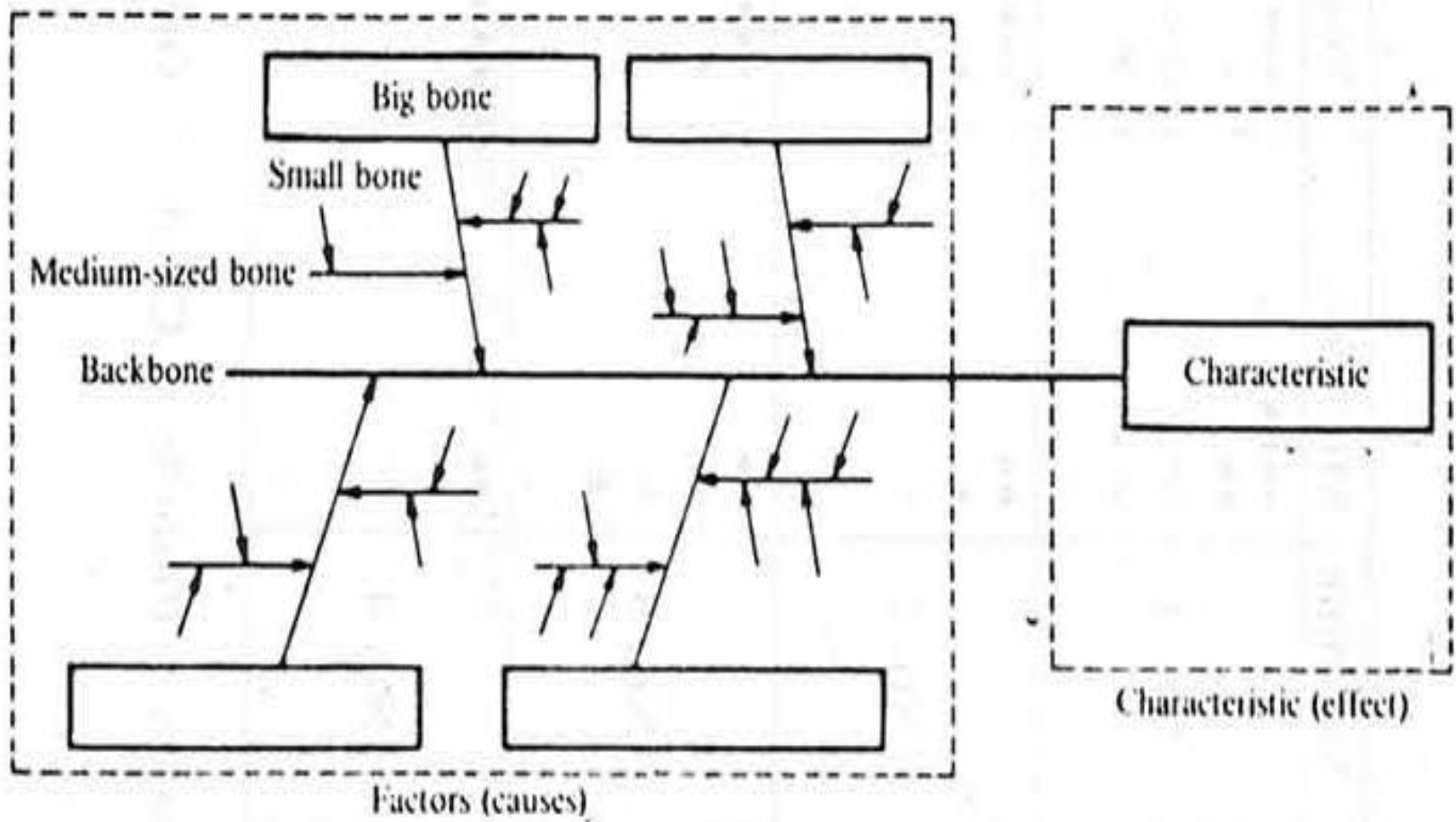


Figure : Cause and Effect Diagram

Cause-and-Effect (cont'd)

- Establish the main causes, or categories, under which other possible causes will be listed.
- Write the main categories your team has selected to the left of the effect box, some above the spine and some below it.
- Draw a box around each category label and use a diagonal line to form a branch connecting the box to the spine.

Cause-and-Effect (cont'd)

Step 4:

For each major branch, identify other specific factors which may be the causes of an effect. Identify as many causes or factors as possible and attach them as sub branches of the major branches.

Fill in detail for each cause. If a minor cause applies to more than one major cause, list it under both.

Cause- and-Effect (cont'd)

Step 5: Identify increasingly more detailed levels of causes and continue organizing them under related causes or categories.

Step 6: Analyze the diagram, this helps you identify causes.

Cause-and-Effect (cont'd)

Example 6.3

The following Figure is a cause and effect diagram for a manual soldering operation. The diagram indicates the effect (the problem is poor solder joints) at the end of the arrow, and the possible causes are listed on the branches leading toward the effect.

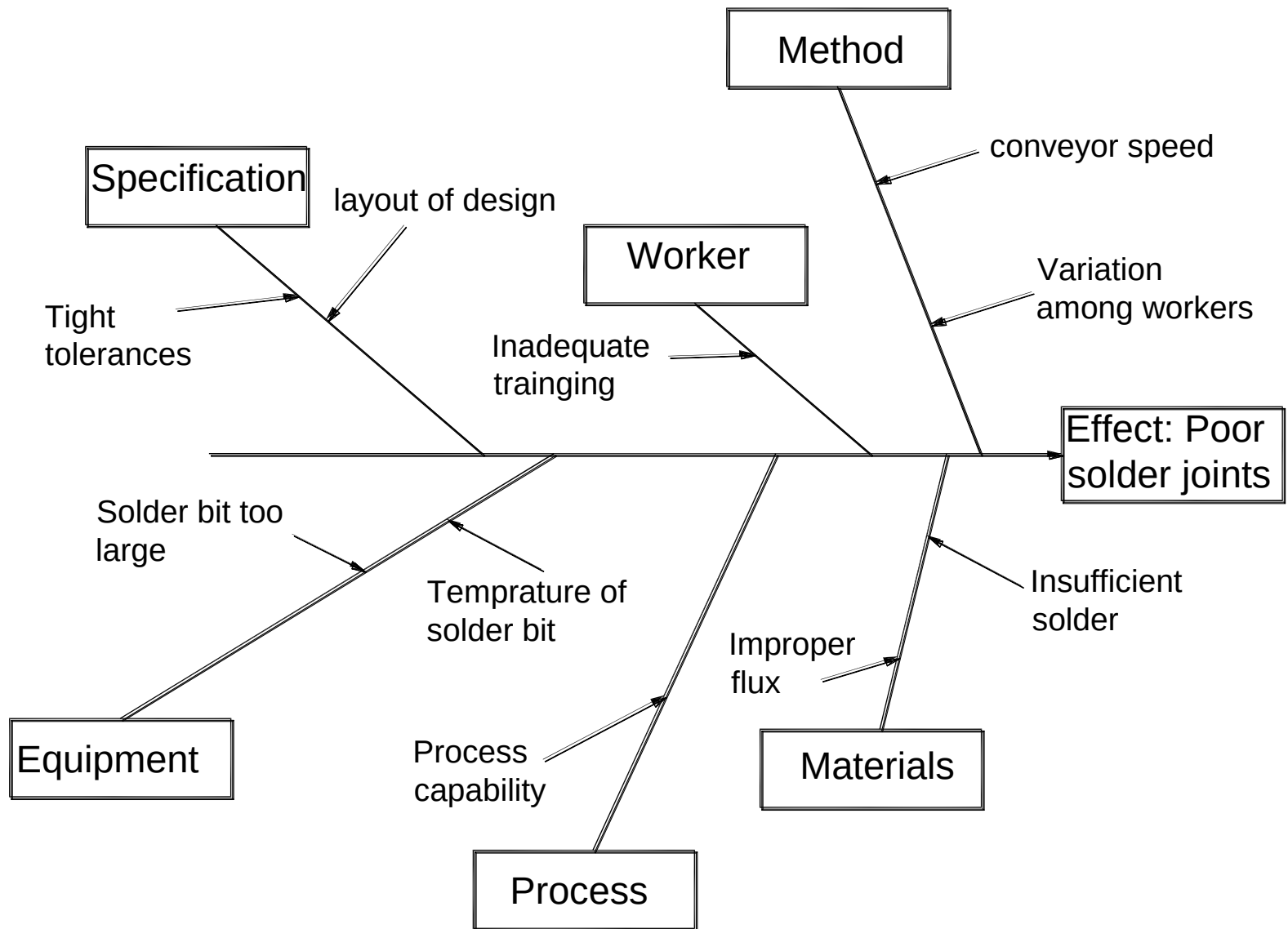


Figure : Cause and Effect Diagram

5. Scatter Diagram

The scatter diagram is a technique used to study the relation of two corresponding variables.

The two variables deal with are:

1. A quality characteristic and a factor affecting it,
2. Two related quality characteristics, or
3. Two factors relating to a single quality characteristic.

Steps to make Scatter diagram

Step 1:

Collect paired data (x, y) , between which you want to study the relations, and arrange the data in a table. It is desirable to have at least 30 pairs of data.

Step 2:

Find the maximum and minimum values for both the x and y . Decide the scales of horizontal and vertical axes so that the both lengths become approximately equal, and then the diagram will be easier to read.

Step 3:

Plot the data on the section paper. When the same data values are obtained from different observations, show these points either by drawing concentric circles, or plot the second point in the immediate vicinity of the first.

Step 4:

Enter all the following necessary items.

1. Title of the diagram
2. Time interval
3. Number of pairs of data
4. Title and units of each axis
5. Name (etc.) of the person who made the diagram.

Example:

A manufacturer of plastic tanks who made them using the blow molding method encountered problems with defective tanks that had thin tank walls. It was suspected that the variation in air pressure, which varied from day to day, was the cause of the non-conforming thin walls.

Table below shows data on blowing air-pressure and percent defective. Let us draw a scatter diagram using this data, according to the steps given above.

Step 1:

As seen in Table below, there are 30 pairs of data.

Table : Variations in Air Pressure

No.	Air Pressure [kgf/cm ²]	Percent Defective[%]	No.	Air pressure [kgf/cm ²]	Percent Defective [%]
1	8.6	0.889	9	9.2	0.895
2	8.9	0.884	10	8.7	0.896
3	8.8	0.874	11	8.4	0.894
4	8.8	0.891	12	8.2	0.864
5	8.4	0.874	13	9.2	0.922
6	8.7	0.886	14	8.7	0.909
7	9.2	0.911	15	9.4	0.905
8	8.6	0.912	16	8.7	0.892

No.	Air Pressure [kgf/cm ²]	Percent Defective[%]	No.	Air pressure [kgf/cm ²]	Percent Defective [%]
17	8.5	0.877	24	8.9	0.908
18	9.2	0.885	25	8.3	0.881
19	8.5	0.866	26	8.7	0.882
20	8.3	0.896	27	8.9	0.904
21	8.7	0.896	28	8.7	0.912
22	9.3	0.928	29	9.1	0.925
23	8.9	0.886	30	8.7	0.872

Step 2:

Blowing air pressure is indicated by y (vertical axis), and percent defective by x (horizontal axis). Then, the maximum value of x : $x_{\max} = 9.4 \text{ (kgf/cm}^2\text{)}$, the minimum value of x : $x_{\min.} = 8.2 \text{ (kgf/cm}^2\text{)}$, the maximum value of y : $y_{\max} = 0.928 \text{ (}\%\text{)}$, the minimum value of y : $y_{\min.} = 0.864 \text{ (}\%\text{)}$.

We mark off the horizontal axis in $0.5 \text{ (kgf/cm}^2\text{)}$ intervals, from $8.0 \text{ to } 9.5 \text{ (kgf/cm}^2\text{)}$, and the vertical axis in $0.01 \text{ (}\%\text{)}$ intervals, from $0.85 \text{ to } 0.93 \text{ (}\%\text{)}$.

Step 3: Plot the data. (See Figure 6.5.)

Step 4: Enter the time interval of the sample obtained (Oct. 1 -Nov. 9), number of samples ($n = 30$), horizontal axis (blowing air-pressure [kgf/cm²]), vertical axis (percent defective [%]), and title of diagram (Scatter diagram of blowing air-pressure and percent defective).

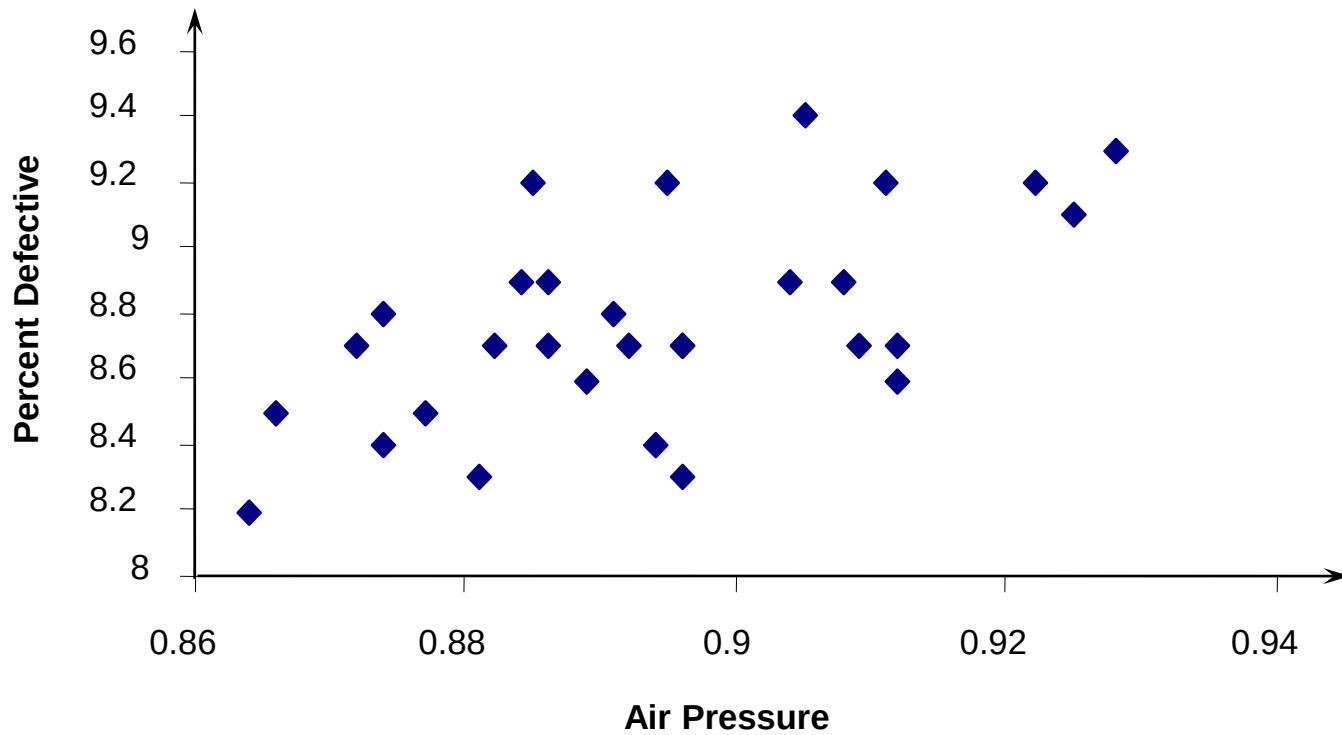


Figure : Scatter Diagram of Blowing Air Pressure and Percent Defective

Types of scatter diagram

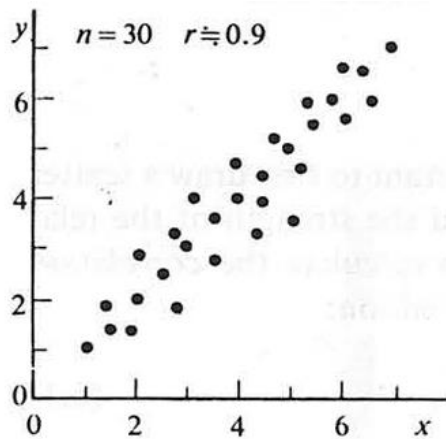


Figure 6.3.1 Positive Correlation

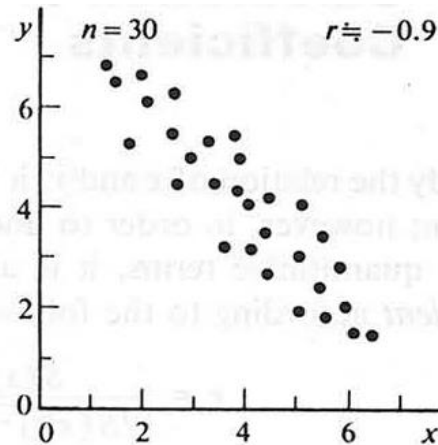


Figure 6.3.4 Negative Correlation

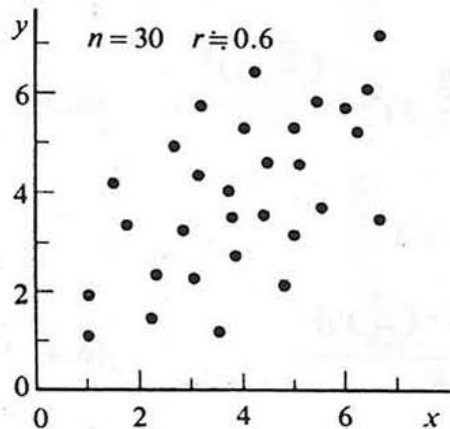


Figure 6.3.2 Positive Correlation
May Be Present

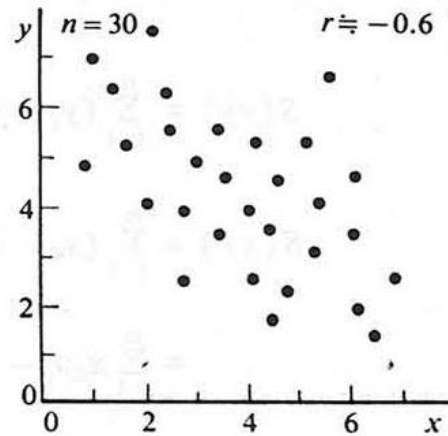


Figure 6.3.5 Negative Correlation
May Be Present

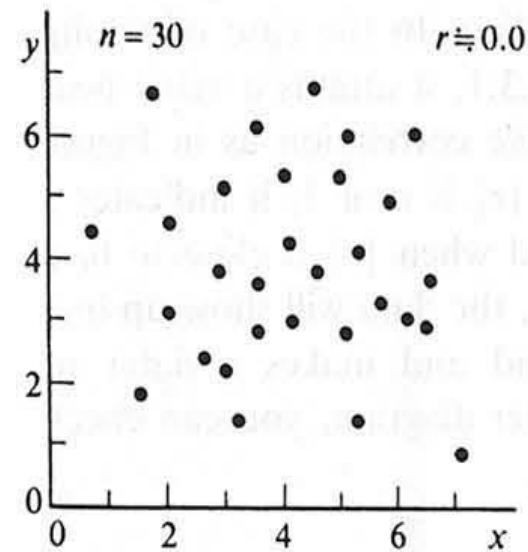


Figure 6.3.3 No Correlation

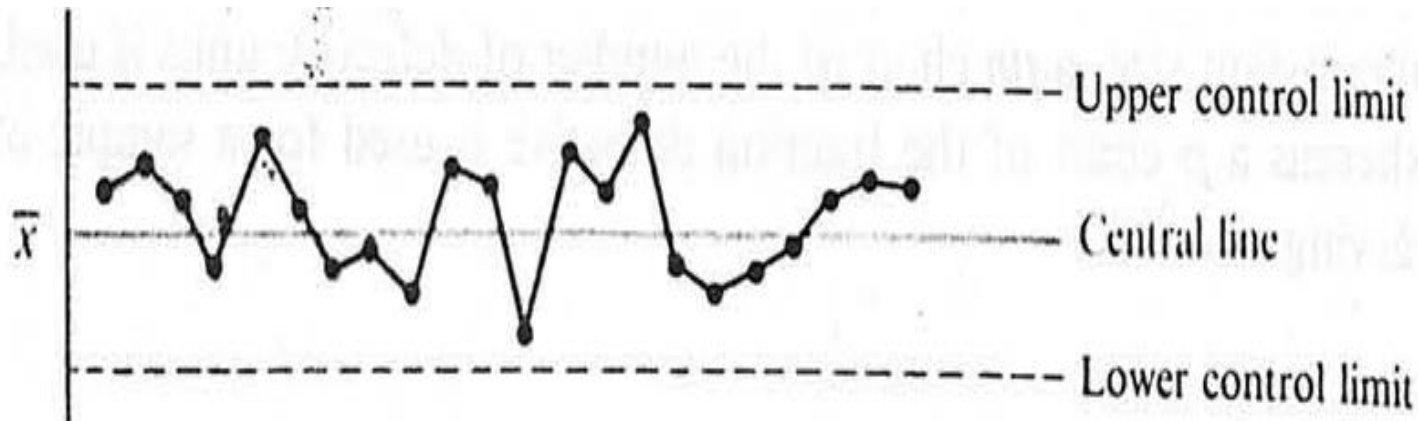
6.Theory of control charts

- ❖ A control chart was first proposed in 1924 by W.A. **Shewhart**, who belonged to the Bell telephone laboratories, with a **view to eliminate an abnormal variation** by distinguishing variations due to **assignable causes** from those due to **chance causes**.

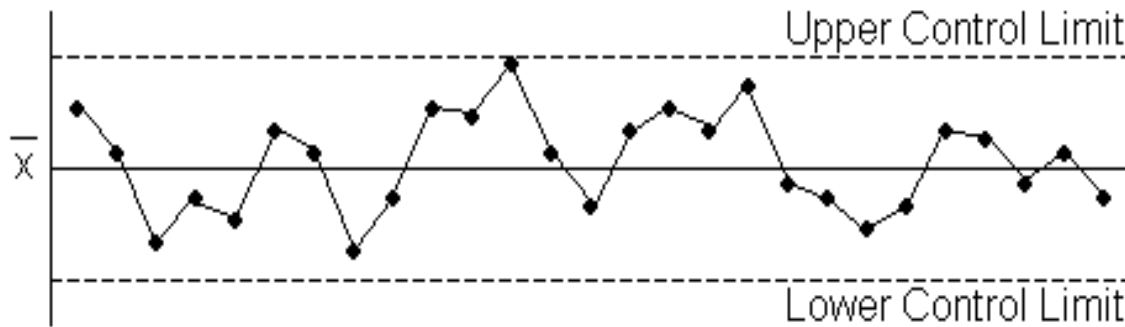
A Control chart is a graphical method for displaying control results and evaluating whether a measurement procedure is in-control or out-of-control.

A control chart consists of:

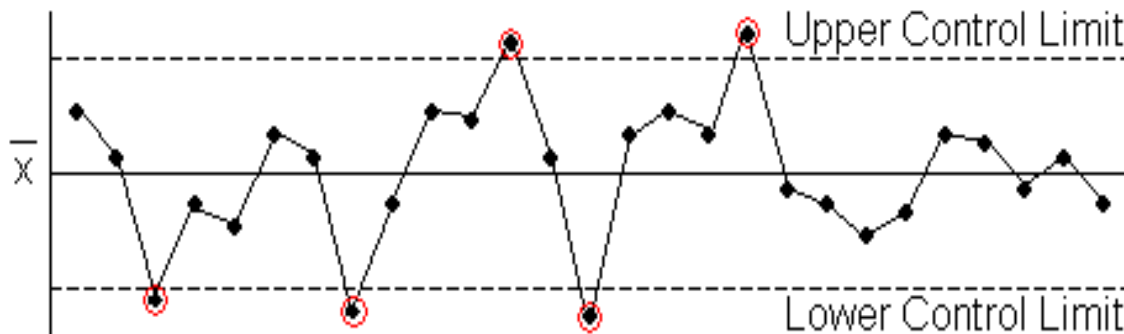
- ❖ A central line
- ❖ Upper control limit
- ❖ Lower control limit and
- ❖ Characteristic values plotted on the chart which represent the state of a process.



If all these values are plotted within the control limits without any particular tendency, the process is regarded as being in the controlled state, however, otherwise it is out of control.



In - Control



Out of Control

Uses of Control charts

The main uses of control charts are:

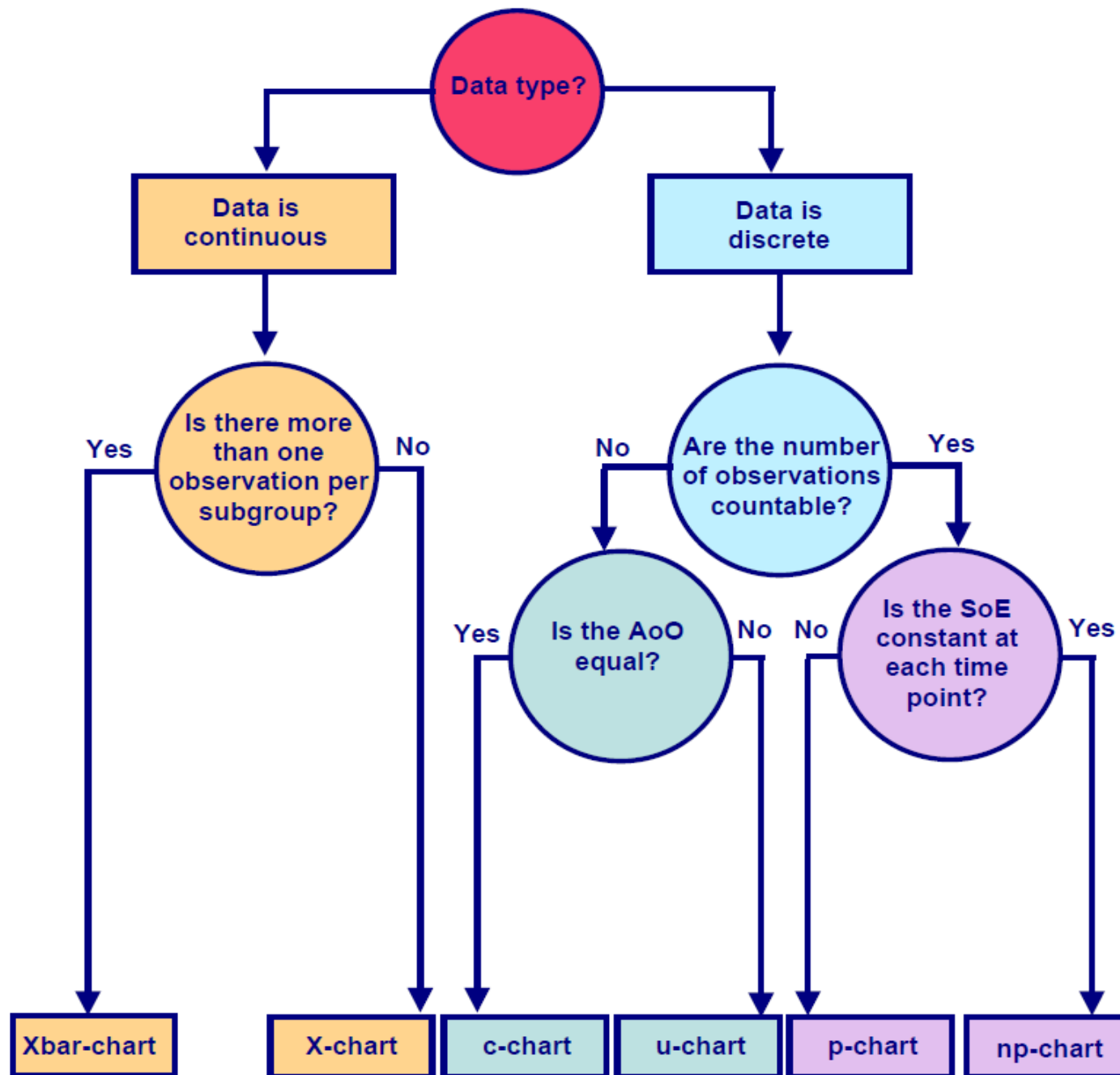
1. It is a proven technique for improving productivity.
2. It is effective in defect prevention.
3. It prevents unnecessary process adjustments.
4. It provides diagnostic information.
5. It provides information about process capability.

Types of control charts

The quality of a product can be evaluated using either an **attribute** of the product or a **variable** measure.

There are two types of control charts:

1. **Control charts for variables**
2. **Control charts for attributes.**



- ❖ A variable measure is a product characteristic that is measured on a continuous scale such as length, weight, volume, pressure, temperature or time.
- ❖ Control charts for attributes summarize the output of a process, or operation, over time. Attributed data have only two values such as good/bad, conforming/non-conforming, or acceptable/not acceptable.

1. Control chart for variables

Two of the most commonly used variable

Control charts are :

- The mean chart or $\bar{X}$ chart, and
- The range or R-chart.

❖ $\bar{X}$ and R Charts

The chart is theoretically based on the normal distribution. It is assumed from the central limit theorem, that the sample means are normally distributed if the process distribution is also normal.

❖ Control charts for variables usually lead to more efficient control procedures and provide more information about process performance than attributes control charts.

$\bar{X}$ and R-chart can be used to:

- ❖ Monitor and control machines and process.
- ❖ Obtain information about specification and manufacturability.
- ❖ Obtain the data about a production run.
- ❖ Supply information to customers of conformance to specifications.

$\bar{X}$ Chart

- ❖ The $\bar{x}$ -bar charts are known as control charts for averages. The $\bar{X}$ -bar chart receives its inputs as the mean of a sample taken from the process under study. Usually the sample will contain four or five observations.

Steps to construct X-bar and R- charts

Step 1 . Collect the data

Step 2. calculate x-bar

Step 3. calculate x-double bar

Step 4. calculate R

Step 5. calculate R-bar

Step 6. calculate the control lines

Step 7. draw the control lines

Step 8. plot the points

Step 9. write the necessary items

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \dots + \bar{x}_k}{k}$$

R = X highest value - X lowest value

$$\bar{R} = \frac{R_1 + R_2 + \dots + R_k}{k}$$

$\bar{x}$ chart

Central line:

$$CL = \bar{\bar{x}}$$

Upper control limit:

$$UCL = \bar{\bar{x}} + A_2 \bar{R}$$

Lower control limit:

$$LCL = \bar{\bar{x}} - A_2 \bar{R}$$

R chart

Central line:

$$CL = \bar{R}$$

Upper control limit:

$$UCL = D_4 \bar{R}$$

Lower control limit:

$$LCL = D_3 \bar{R}$$

Tabulation of factor A_2 for $\bar{X}$ charts

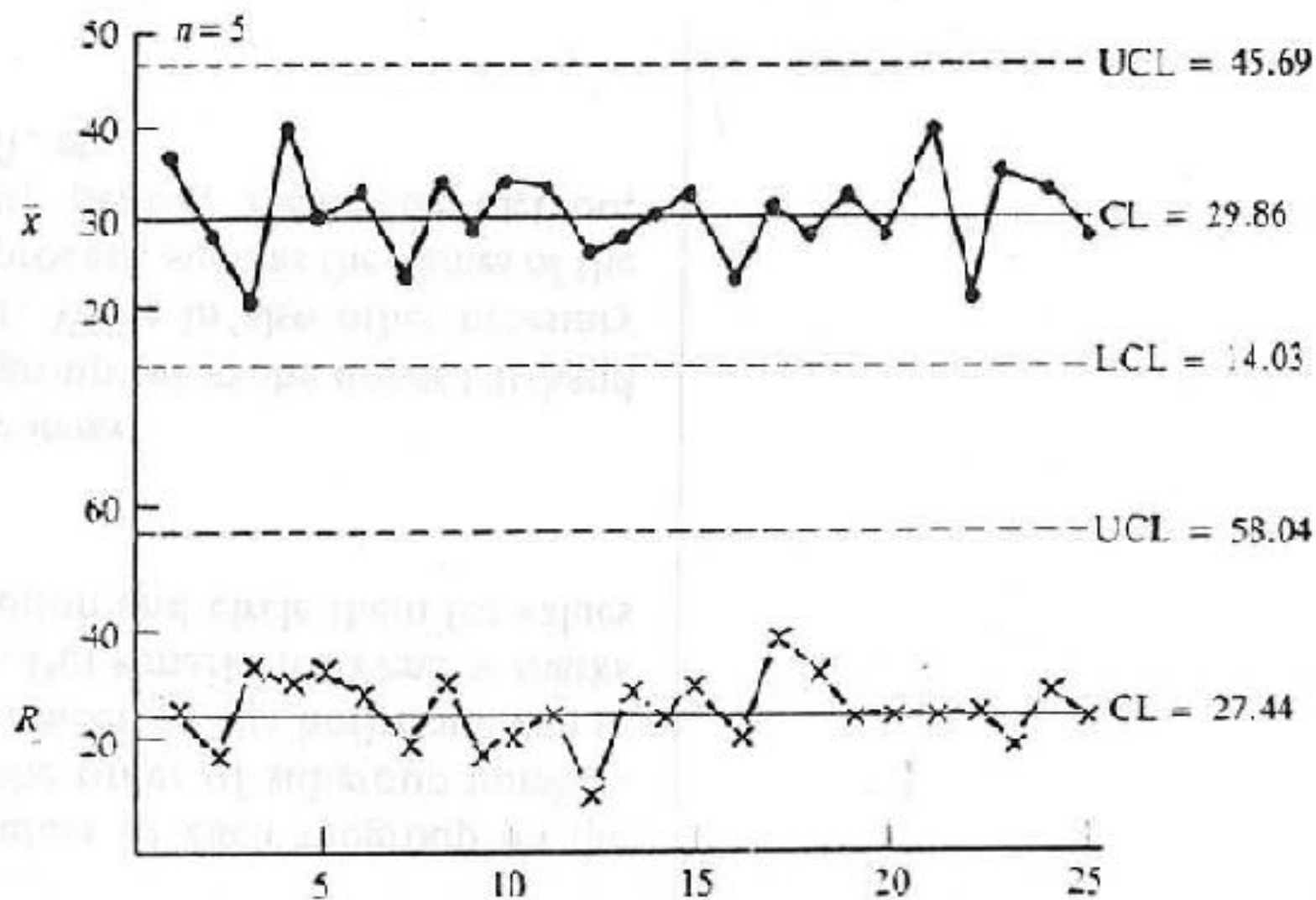
n	2	3	4	5	6	7	8	9
A_2	1.880	1.023	0.729	0.577	0.483	0.419	0.373	0.337

Table of D3 and D4 Values

n	2	3	4	5	6	7	8	9
D_3	0	0	0	0	0	0.076	0.136	0.184
D_4	3.267	2.574	2.282	2.114	2.004	1.924	1.864	1.816

3	19	11	16	11	44	101	20.2	33
4	29	29	42	59	38	197	39.4	30
5	28	12	45	36	25	146	29.2	33
6	40	35	11	38	33	157	31.4	29
7	15	30	12	33	26	116	23.2	21
8	35	44	32	11	38	160	32.0	33
9	27	37	26	20	35	145	29.0	17
10	23	45	26	37	32	163	32.6	22
11	28	44	40	31	18	161	32.2	26
12	31	25	24	32	22	134	26.8	10
13	22	37	19	47	14	139	27.8	33
14	37	32	12	38	30	149	29.8	26
15	25	40	24	50	19	158	31.6	31
16	7	31	23	18	32	111	22.2	25
17	38	0	41	40	37	156	31.2	41
18	35	12	29	48	20	144	28.8	36
19	31	20	35	24	47	157	31.4	2
20	12	27	38	40	31	148	29.6	28
21	52	42	52	24	25	195	39.0	28
22	20	31	15	3	28	97	19.4	28
23	29	47	41	32	22	171	34.2	25
24	28	27	22	32	54	163	32.6	32
25	42	34	15	29	21	141	28.2	27
Total							746.6	686
Average							$\bar{x} =$ 29.86	$\bar{R} =$ 27.44

Collect approximately 100 data. Divide them into 20 to 25 subgroups with 4 to 5 in each, making each of them uniform within the subgroup. Fill a data sheet



2. Control chart for attributes

Attribute control charts arise when items are compared with some standard and then are classified as to whether they meet the standard or not.

Four types of attribute control charts:

1. P-chart
2. np-chart
3. C-chart
4. U-chart

Discrete value

pn chart (number of defective units)

p chart (fraction defective)

c chart (number of defects)

u chart (number of defects per unit)

i) P-Chart

- ❖ P-chart measures the proportion of defective products in a batch, lot or shipment of products.
- ❖ With the P-chart, a sample is taken periodically from the process and the proportion of defective items in the sample is determined to see if it falls within the control limits in the chart.

Since a P-chart employs a discrete, attribute measure (defective items) it is theoretically based on the binomial distribution. However, as the sample size gets larger, the normal distribution can be used to approximate the binomial distribution. It is used when the subgroups are not of equal size. The np chart is used in the more limited case of equal subgroups.

Steps in Constructing a **P-Chart**

Step 1: Collect data and organize in subgroups

Step 2: Determine the fraction defective. To calculate this value use:

$$p = \frac{np}{n}$$

Where p = fraction defective (non-conformity)

np = number of defective products in subgroup

n = number of inspected products in subgroup

Step 3: Determine the process average (*the ratio of number of defective products in all of the subgroups divided by total number of products*):

$$\bar{p} = \frac{np_1 + np_2 + np_3 + \dots + np_k}{n_1 + n_2 + n_3 + \dots + n_k}$$

Where:

np_1 = Number of defective products in 1st subgroup

n_1 = Number of products in the first subgroup

n_k = Number of products in the kth subgroup

Step 4: Determine the standard deviation

$$\sigma = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

Step 5: Determine the control limits (UCL,LCL)

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} \quad \text{OR} \quad UCL = \bar{p} + 3\sigma$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} \quad \text{OR} \quad LCL = \bar{p} - 3\sigma$$

**Step 6: Plot the centerline, the LCL and UCL,
and the process measurements.**

np-Chart

- The "p" and "np" charts are very similar.
- The p chart graphs the fraction defective.
- The np chart displays the actual number of non-conforming products. The number of non-conforming or defective is the product of the sample size and the fraction defective.

Steps in Constructing **nP-Chart**

Step 1: Collect data and organize in subgroups

Step 2: Determine the fraction defective. To calculate this value use:

$$p = \frac{np}{n}$$

Where p = the fraction defective

np = number of defectives

n = size of sample.

Step 3: Determine the np process average:

$$n\bar{p} = \frac{np_1 + np_2 + np_3 + \dots + np_k}{k}$$

Where:

$n\bar{p}$ = Defective process average

np_1 = Number of defectives in first subgroup sample

k = Number of subgroups

Step 4: Determine the standard deviation

$$\sigma = \sqrt{n \bar{p}(1 - \bar{p})}$$

Step 5: Determine the control limits (UCL,LCL)

$$UCL = n \bar{p} + 3\sqrt{n \bar{p}(1 - \bar{p})} \quad \text{OR} \quad UCL = n \bar{p} + 3\sigma$$

$$LCL = n \bar{p} - 3\sqrt{n \bar{p}(1 - \bar{p})} \quad \text{OR} \quad LCL = n \bar{p} - 3\sigma$$

**Step 6: Plot the centerline, the LCL and UCL,
and the process measurements.**

Example

Frozen orange juice concentrate is packed in 6-oz cardboard cans. These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. Such a nonconforming can has an improper seal on either the side seam or the bottom panel. Set up a control chart to improve the fraction nonconforming cans produced by this machine.

Sample size n=50

Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$	Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$
1	12	0.24	17	10	0.20
2	15	0.30	18	5	0.10
3	8	0.16	19	13	0.26
4	10	0.20	20	11	0.22
5	4	0.08	21	20	0.40
6	7	0.14	22	18	0.36
7	16	0.32	23	24	0.48
8	9	0.18	24	15	0.30
9	14	0.28	25	9	0.18
10	10	0.20	26	12	0.24
11	5	0.10	27	7	0.14
12	6	0.12	28	13	0.26
13	17	0.34	29	9	0.18
14	12	0.24	30	6	0.12
15	22	0.44		347	$\bar{p} = 0.2313$
16	8	0.16			

Solution

30 samples contain $\sum_{i=1}^{30} D_i = 347$ nonconforming cans.

$$\bar{p} = \frac{\sum_{i=1}^m D_i}{mn} = \frac{347}{(30)(50)} = 0.2313$$

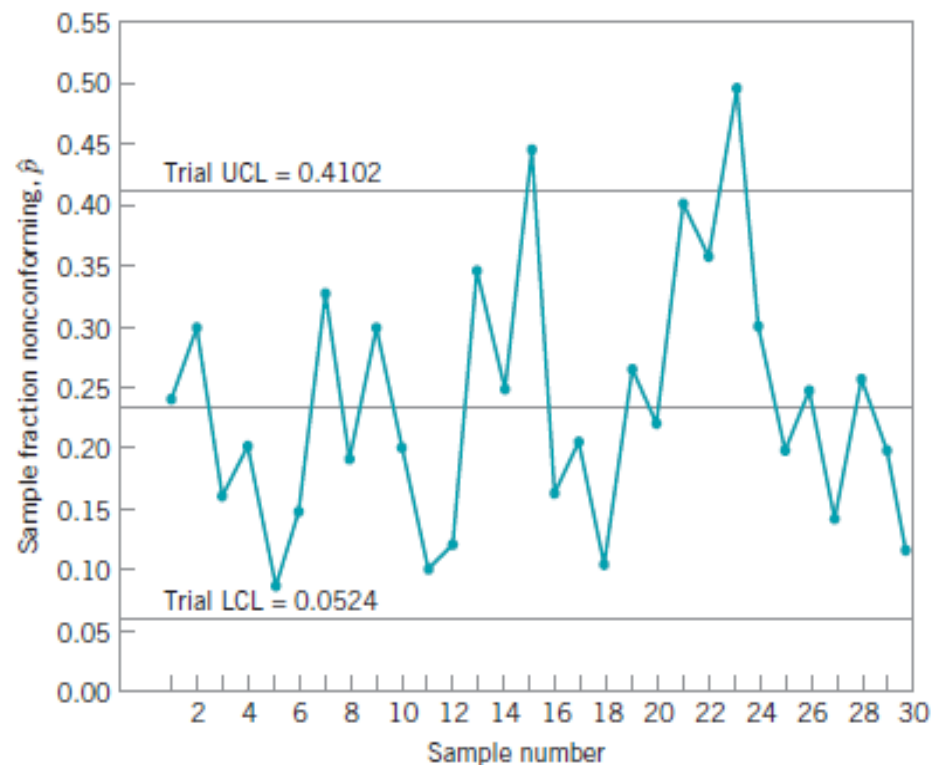
$$\begin{aligned}\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} &= 0.2313 \pm 3\sqrt{\frac{0.2313(0.7687)}{50}} \\ &= 0.2313 \pm 3(0.0596) \\ &= 0.2313 \pm 0.1789\end{aligned}$$

Therefore,

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2313 + 0.1789 = 0.4102$$

and

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2313 - 0.1789 = 0.0524$$



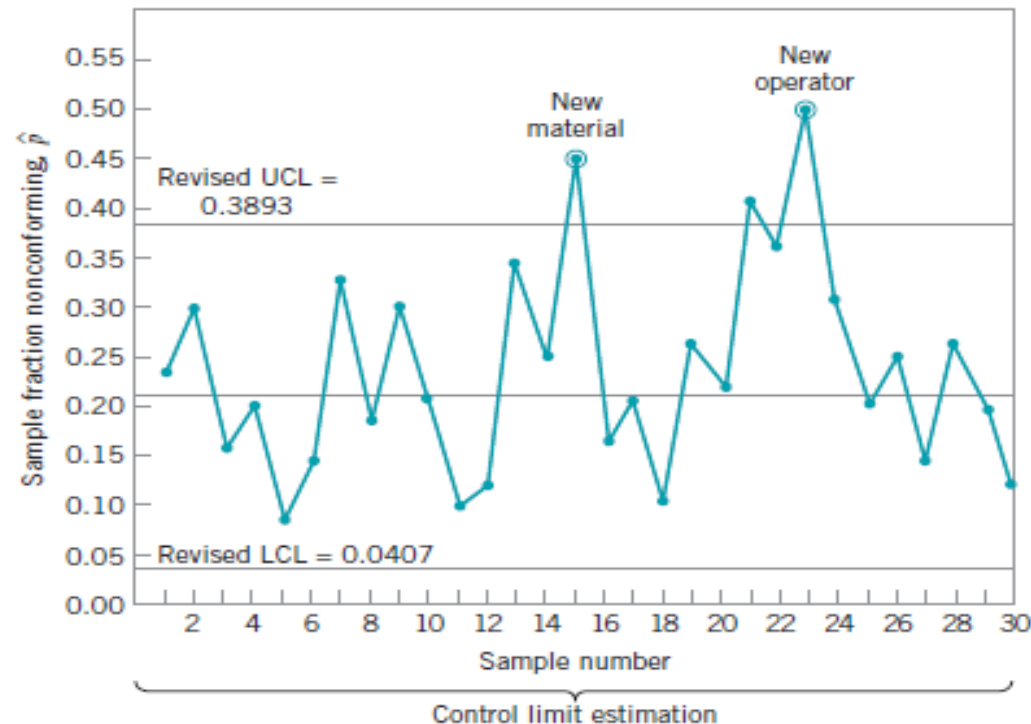
Cont...

- We note that **two points**, those from samples **15** and **23**, plot above the upper control limit, so the **process is not in control**.
- These points must be investigated to see whether an assignable cause can be determined.
- The revised results are;

$$\bar{p} = \frac{301}{(28)(50)} = 0.2150$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2150 + 3\sqrt{\frac{0.2150(0.7850)}{50}} = 0.3893$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2150 - 3\sqrt{\frac{0.2150(0.7850)}{50}} = 0.0407$$



Cont...

- Set up an np control chart for the orange juice concentrate can process in the above Example is;

$$\bar{p} = 0.2313 \quad n = 50$$

Therefore, the parameters of the np control chart would be

$$\begin{aligned} \text{UCL} &= n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \\ &= 50(0.2313) + 3\sqrt{(50)(0.2313)(0.7687)} \\ &= 20.510 \end{aligned}$$

$$\text{Center line} = n\bar{p} = 50(0.2313) = 11.565$$

$$\begin{aligned} \text{LCL} &= n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} \\ &= 50(0.2313) - 3\sqrt{(50)(0.2313)(0.7687)} \\ &= 2.620 \end{aligned}$$

C - chart

C-chart measures the number of nonconformities (defectives) per "unit" and is denoted by c . This "unit" is commonly referred to as an inspection unit and may be "per day" or "per square foot" of some other predetermined sensible rate.

- The c-chart are also known as the control charts for defects per unit.
- Theoretically these charts are used in situations where the opportunities for defects to occur in an item are large.
- In other words, these charts are used to control the number of defects in the item.

- Examples are:

1. The number of surface scratches on the printed circuit boards.
2. The number of mechanical defects per lot of a given quantity of units.

❖ The c-chart is based on the Poisson distribution. The Poisson distribution is usually used to describe the number of arrivals per time. Here, the opportunity for the occurrence of an event, n is large, but the probability of each occurrence, p is quite small.

Step 1: The mean $\bar{C}$ of the Poisson distribution is given by:

$$\bar{C} = \frac{\text{Total number of defects}}{\text{Total number of items}}$$

Step 2: Determine the standard deviation:

$$\sigma = \sqrt{\bar{C}}$$

Step 3: calculate upper and lower control limits

$$UCL = \bar{C} + 3\sqrt{\bar{C}} \quad \text{OR} \quad UCL = \bar{C} + 3\sigma$$

$$LCL = \bar{C} - 3\sqrt{\bar{C}} \quad \text{OR} \quad LCL = \bar{C} - 3\sigma$$

Step 4: Plot the control limits and the points.

Example

Data on the Number of Nonconformities in Samples of 100 Printed Circuit Boards

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	21	14	19
2	24	15	10
3	16	16	17
4	12	17	13
5	15	18	22
6	5	19	18
7	28	20	39
8	20	21	30
9	31	22	24
10	25	23	16
11	20	24	19
12	24	25	17
13	16	26	15

Solution

Since the 26 samples contain 516 total nonconformities, we estimate $\bar{c}$ by

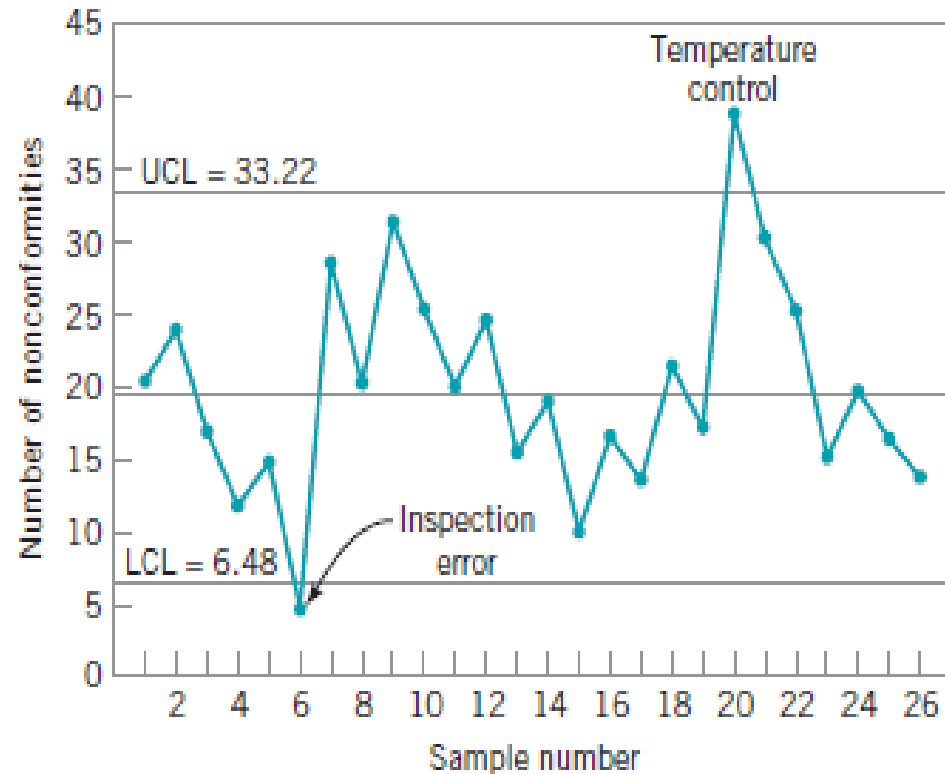
$$\bar{c} = \frac{516}{26} = 19.85$$

Therefore, the trial control limits are given by

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 19.85 + 3\sqrt{19.85} = 33.22$$

$$\text{Center line} = \bar{c} = 19.85$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 19.85 - 3\sqrt{19.85} = 6.48$$



u-Chart

There are cases where the constant sample lot sizes, as used for the c-chart, are not feasible. In those instances the u-chart is used. The u-chart measures the number of defects per product. It is similar to the c-chart, except that the number of defects are expressed on a per unit basis.

The u Chart is used when it is not possible to have an inspection unit of a fixed size (e.g., 12 defects counted in one square foot), rather the number of nonconformities is per *inspection unit*.

Steps in constructing a u-Chart

Step1:

Find the number of nonconformities, $c(i)$ and the number of inspection units, $n(i)$, in each sample i .

Step 2:

Compute $u(i) = c(i)/n(i)$

Where $U(i)$ = defects per unit

C = number of defects discovered in a lot

n = the number of inspection units

Step 3:

Determine the centerline of the u chart:

$$\bar{U} = \frac{\text{total nonconformance in } k \text{ sub - group}}{\text{total number of inspection units}}$$

$$\bar{U} = \frac{c(1) + c(2) + \dots c(k)}{n(1) + n(2) + \dots n(k)}$$

Step 4:

The u chart has individual control limits for each subgroup i.

$$UCL = \bar{U} + 3\sqrt{\frac{\bar{U}}{n(i)}}$$

$$LCL = \bar{U} - 3\sqrt{\frac{\bar{U}}{n(i)}}$$

Step 5:

Plot the centerline, $\bar{\bar{x}}$, the individual LCL and UCL, and the process measurements, $u(i)$.

Step 6:

Interpret the control chart.

Example

Data on Number of Shipping Errors in a Supply Chain Network

Sample Number (week), i	Sample Size, n	Total Number of Errors (Nonconformities), x_i	Average Number of Errors (Nonconformities) per Unit, $u_i = x_i/n$
1	50	2	0.04
2	50	3	0.06
3	50	8	0.16
4	50	1	0.02
5	50	1	0.02
6	50	4	0.08
7	50	1	0.02
8	50	4	0.08
9	50	5	0.10
10	50	1	0.02
11	50	8	0.16
12	50	2	0.04
13	50	4	0.08
14	50	3	0.06
15	50	4	0.08
16	50	1	0.02
17	50	8	0.16
18	50	3	0.06
19	50	7	0.14
20	50	4	0.08
		74	1.48

Solution

- We estimate the number of errors (nonconformities) per unit (shipment) to be;

$$\bar{u} = \frac{\sum_{i=1}^{20} u_i}{20} = \frac{1.48}{20} = 0.0740$$

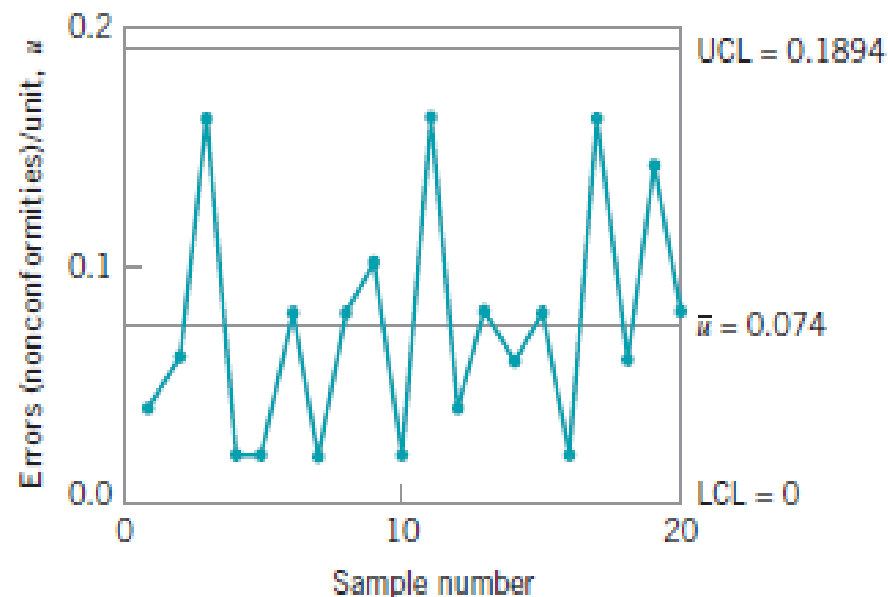
Therefore, the parameters of the control chart are

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 0.0740 + 3\sqrt{\frac{0.0740}{50}} = 0.1894$$

$$\text{Center line} = \bar{u} = 0.074$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 0.0740 - 3\sqrt{\frac{0.0740}{50}} = -0.0414$$

Since the $LCL < 0$, we would set $LCL = 0$ for the u chart.



Type of Control Chart	Upper Control Limit (UCL), Central Line (CL), Lower Control Limit (LCL)
$\bar{x}$	$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + A_2 \bar{R} \\ \text{CL} &= \bar{\bar{x}} \\ \text{LCL} &= \bar{\bar{x}} - A_2 \bar{R} \end{aligned}$
R	$\begin{aligned} \text{UCL} &= D_4 \bar{R} \\ \text{CL} &= \bar{R} \\ \text{LCL} &= D_3 \bar{R} \end{aligned}$
s	$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + 2.66 \bar{R}_s \\ \text{CL} &= \bar{\bar{x}} \\ \text{LCL} &= \bar{\bar{x}} - 2.66 \bar{R}_s \end{aligned}$
pn	$\begin{aligned} \text{UCL} &= \bar{p}n + 3 \sqrt{\bar{p}n(1 - \bar{p})} \\ \text{CL} &= \bar{p}n \\ \text{LCL} &= \bar{p}n - 3 \sqrt{\bar{p}n(1 - \bar{p})} \end{aligned}$
p	$\begin{aligned} \text{UCL} &= \bar{p} + 3 \sqrt{\bar{p}(1 - \bar{p})/n} \\ \text{CL} &= \bar{p} \\ \text{LCL} &= \bar{p} - 3 \sqrt{\bar{p}(1 - \bar{p})/n} \end{aligned}$
c	$\begin{aligned} \text{UCL} &= \bar{c} + 3 \sqrt{\bar{c}} \\ \text{CL} &= \bar{c} \\ \text{LCL} &= \bar{c} - 3 \sqrt{\bar{c}} \end{aligned}$
u	$\begin{aligned} \text{UCL} &= \bar{u} + 3 \sqrt{\bar{u}/n} \\ \text{CL} &= \bar{u} \\ \text{LCL} &= \bar{u} - 3 \sqrt{\bar{u}/n} \end{aligned}$

7. Defect concentration diagram

- A defect concentration diagram is a picture of the unit, showing all relevant views. Then the various types of defects are drawn on the picture, and the diagram is analyzed to determine whether the location of the defect on the unit conveys any useful information about the potential causes of the defects.

- Most of the time, surface finish defects occur on most products which is identified by a different color or shape, from the inspection of the diagram defect may be due to improper material handling.